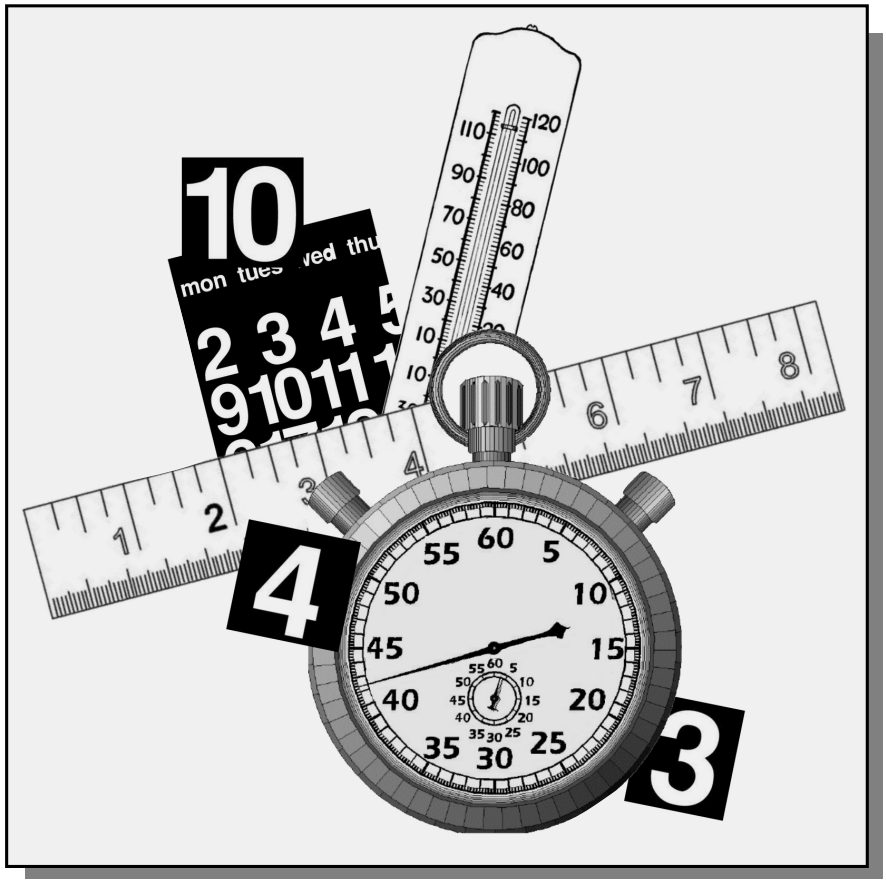
Intermediate Math Skills



Chapter 4

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Chapter 4

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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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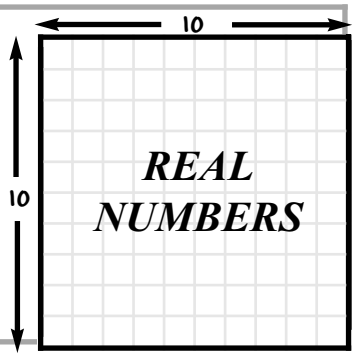
Intermediate Math Skills



Lesson 1

Fundamentals

Chapter 4



Section 1

Lessons 1-5



If you recall, Professor K explained the real number system to you in previous lessons. As a quick review of the real number system Professor K offers the ***Venn diagram*** in **Figure 1.1**.



Professor K

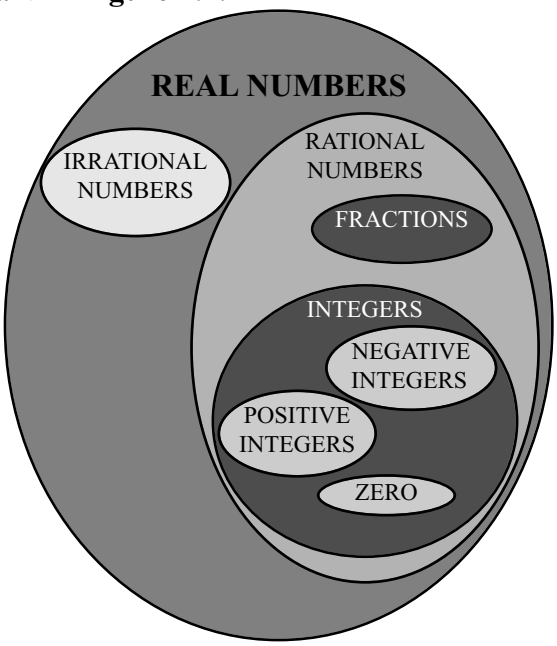


Figure 1.1

VOCABULARY

Venn diagram: a pictorial representation of sets and subsets
Sets: a collection of objects
Subsets: a collection of objects in which all the members of one collection are contained within another set
Family tree: a pictorial representation of the members of a family
Integers: set of numbers consisting of whole numbers and the negative of the counting numbers: …-3, -2, -1, 0, 1, 2, 3, …

In a Venn diagram such as the one in **Figure 1.1**, smaller ***sets*** that are contained within larger sets are shown within the larger set. Venn diagrams are a pictorial representation of sets and ***subsets***. You have probably seen a ***family tree***. Family trees are like a larger set with many subsets, but in reverse order. Look at the family tree shown in **Figure 1.2**.

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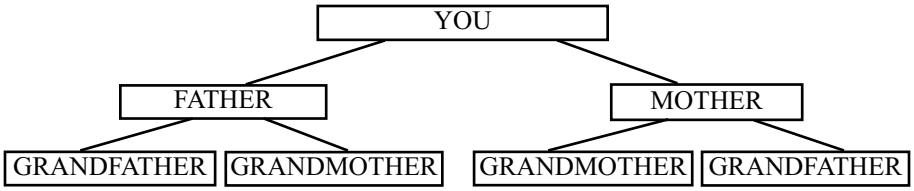


Figure 1.2

The set of real numbers and its subsets can be shown as a family of numbers. Notice in **Figure 1.3** how Professor K illustrates sets and subsets. Notice how the diagram in **Figure 1.3** is another way to present the subsets (or family) of the Real Numbers as presented in **Figure 1.1**.

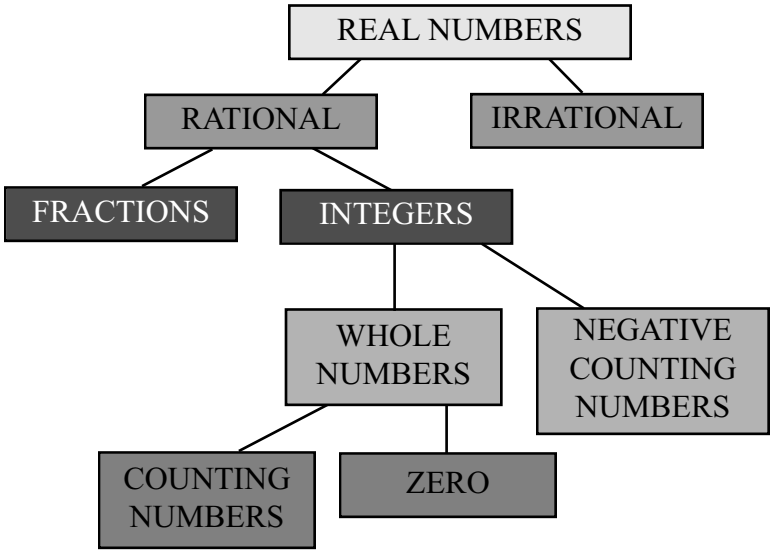


Figure 1.3

You will remember that rational numbers can be written exactly as fractions. Notice the root word for *rational* is *ratio*. (A ratio can be written in fraction form.) Other numbers within the set of rational numbers are known as ***integers***. You studied *counting numbers* (1, 2, 3, 4, …) and *whole numbers* (0, 1, 2, 3, 4, …) in previous lessons. Integers are extensions of whole numbers. *Whole numbers* are always *positive counting numbers* or *zero*, but integers can be positive counting numbers, negative counting numbers or zero (…-3, -2, -1, 0, 1, 2, 3, …).

The other branch of the family tree of real numbers is *irrational numbers*. These numbers are those that cannot be expressed exactly as fractions, terminating decimals or repeating decimals. Numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$ and π are examples of irrational numbers.

Look at **Example Set 1** as Professor K identifies each number.

Example Set 1

Professor K will name all the sets that include the number shown. (Real, Rational, Integers, Whole numbers, or Irrational.)

a) 4

a) The number 4 is a positive integer. Therefore, 4 is a member of the sets of whole numbers (0, 1, 2, 3, 4, …), the integers (…-3, -2, -1, 0, 1, 2, 3, …), the rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

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b) -10

b) The number, *-10* is a negative counting number or integer. Therefore, -10 is a member of the integers (…-3, -2, -1, 0, 1, 2, 3, …), the rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

c) $^1/_5$

c) The number, $^1/_5$ is a member of the sets of the rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

In **Example Set 2** Professor K identifies more numbers.



Example Set 2

Professor K will name all the sets that include the number shown (Real, Rational, Integers, Whole numbers, or Irrational).

a) $^{25}/_5$

a) The number, $^{25}/_5$ is a positive integer, because $^{25}/_5 = 5$. Therefore, $^{25}/_5$ or 5 is a member of the sets of the whole numbers (0, 1, 2, 3, 4, …), the integers (…-3, -2, -1, 0, 1, 2, 3, …), the rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

b) $-^{81}/_9$

b) The number, $-^{81}/_9$ is a negative integer, because $-^{81}/_9 = -9$. Therefore, $-^{81}/_9$ or -9 is a member of the sets of the integers (…-3, -2, -1, 0, 1, 2, 3, …), the rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

c) $^2/_{27}$

c) The number, $^2/_{27}$ is a member of the sets of rational numbers; numbers that can be expressed exactly as fractions with an integer in the numerator and denominator ($^1/_2$, $^5/_7$, $^3/_1$, $^{16}/_4$) and the real numbers.

d) $\sqrt{11}$

d) The number, $\sqrt{11}$ is an irrational number, because $\sqrt{11}$ cannot be expressed exactly as a fraction or a repeating or terminating decimal.

e) $\sqrt{17}$

e) The number, $\sqrt{17}$ is an irrational number, because $\sqrt{17}$ cannot be expressed exactly as a fraction or a repeating or terminating decimal.

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Show Professor K that you understand how to determine if a number is real, rational, irrational, an integer, or whole number by working the activities for this lesson.

Life Principle



"A person is the sum of his
or her life choices."

—Anonymous

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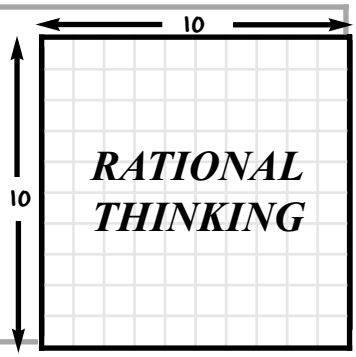
Intermediate Math Skills



Lesson 2

Radicals

Chapter 4



Section 1

Lessons 1-5





Often, when simplifying numerical or algebraic expressions, a number written in radical form (an expression containing a radical sign) will end up in the denominator of the fractional answer. Just as the denominator of a fraction is not supposed to be zero ($^{x}/_{0}$ = undefined), an answer is not in proper form if the denominator contains an expression written in radical form.



RULE

For a fraction to be written in proper form, the denominator of the fraction must be a rational number. The denominator cannot contain a number written in radical form.

$$\frac{5}{\sqrt{4}} = \frac{5}{\sqrt{2^2}} = \frac{5}{2}$$

$$\frac{7}{\sqrt{9}} = \frac{7}{\sqrt{3^2}} = \frac{7}{3}$$

$$\frac{3}{\sqrt{25}} = \frac{3}{\sqrt{5^2}} = \frac{3}{5}$$

VOCABULARY

Rationalizing the denominator: method used to remove the radical sign from the denominator of a fraction
Evaluated: calculated the answer to a problem; simplified a mathematical expression

In this lesson Professor K will show you how to accomplish the mathematical operation called ***rationalizing the denominator***. Follow the examples closely so that you, too, can rationalize the denominator of fractions that contain irrational numbers.

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Example Set 1

Step 1 $\dfrac{5[3+2(4-1)]}{2[\sqrt{2}(5-2)]}$

By looking at the expression to be ***evaluated***, you can see that the numerical expression is written in fraction form. You must evaluate the expression in the numerator and then the denominator separately and then bring them together to evaluate or simplify the expression.

Step 2

$5[3 + 2(4 - 1)]$
$5[3 + 2(3)]$
$5[3 + 6]$
$5[9]$
45

Notice the numerator has two sets of grouping symbols. [( )] You must use PEMDAS (Order of Operations) as shown, going to the innermost set of symbols ( ) first and working your way outward. The evaluation of the numerator is 45.

Step 3 $2[\sqrt{2}(5-2)]$

Now you must evaluate the algebraic expression in the denominator.

Step 4 $2[\sqrt{2}(3)]$

Once again, two sets of grouping symbols are seen and PEMDAS rules are followed as in the numerator above.

Step 5

$2 \bullet 3\sqrt{2}$
$6\sqrt{2}$

The Commutative Property is used to place the 2 and 3 together for multiplication. The evaluation of the denominator is $6\sqrt{2}$.

Step 6

$\dfrac{45}{6\sqrt{2}}$
$\dfrac{\cancel{3} \bullet 15}{2 \bullet \cancel{3}\sqrt{2}}$
$\dfrac{15}{2\sqrt{2}}$

You now must bring the numerator and denominator together in the same fraction again. The fraction is divided into its factors and reduced as shown to obtain the final evaluation of the original expression. This answer is not in proper form because the denominator contains an irrational number. The denominator must be rationalized.



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Professor K will rationalize the denominator in **Example Set 2**.

Example Set 2

Rationalize the denominator in the following algebraic expression.

Step 1 $\dfrac{15}{2\sqrt{2}}$

By looking at the expression to be rationalized, you can see that the numerical expression is written in fraction form and the denominator contains an irrational number. You must rationalize the denominator of the expression.

Step 2 $\dfrac{15}{2\sqrt{2}} \bullet \dfrac{\sqrt{2}}{\sqrt{2}}$

Remember that we can multiply the numerator and denominator of a fraction by the same number and not change the value of the fraction. Professor K uses this fact in the first calculation here. Professor K chooses the $\sqrt{2}$ as the multiplier so the $\sqrt{2}$ in the denominator would multiply out ($\sqrt{2} \bullet \sqrt{2} = 2$).

Step 3 $\dfrac{15 \bullet \sqrt{2}}{2(\sqrt{2} \bullet \sqrt{2})}$

The next calculation shows the grouping of the factors for multiplication.

Step 4 $\dfrac{15 \bullet \sqrt{2}}{2 \bullet 2}$

This calculation shows the result of $\sqrt{2} \bullet \sqrt{2}$ multiplied by 2.

Step 5 $\dfrac{15\sqrt{2}}{4}$

Finally, Professor K has performed the calculations and he has completed the rationalization of the denominator. The fraction is now in proper form.

Example Set 3

Rationalize the denominator in the following fractions.

a) $\dfrac{25}{\sqrt{3}}$

$\dfrac{25}{\sqrt{3}} \bullet \dfrac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{25\sqrt{3}}{3}$

This expression is not in proper form. Professor K must rationalize the denominator. Professor K chooses $\sqrt{3}$ as a multiplier of the numerator and denominator to rationalize the denominator. Professor K uses the rules for multiplying fractions and obtains the answer with a rationalized denominator.



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b) $\dfrac{\sqrt{5}}{\sqrt{7}}$

$\dfrac{\sqrt{5}}{\sqrt{7}} \bullet \dfrac{\sqrt{7}}{\sqrt{7}}$

$\dfrac{\sqrt{35}}{7}$

This expression is not in proper form. Professor must rationalize the denominator. Professor K chooses $\sqrt{7}$ as a multiplier of the numerator and denominator to rationalize the denominator. Professor K points out that he used the radical that appeared in the denominator, not the numerator. Professor K uses the rules for multiplying fractions and obtains the answer with a rationalized denominator.

In **Example Sets 1 and 2**, Professor K used the rule shown below for multiplying radicals together.



RULE

To multiply numbers together that contain radicals, multiply the numbers outside the radical symbol together and multiply the numbers contained within the radical symbols together.



$$4\sqrt{2} \bullet 3\sqrt{5} = (4 \bullet 3)\sqrt{(2 \bullet 5)} = 12\sqrt{10}$$

$$\sqrt{3} \bullet \sqrt{11} = \sqrt{3 \bullet 11} = \sqrt{33}$$

$$2\sqrt{3} \bullet 4\sqrt{5} \bullet 3\sqrt{7} = (2 \bullet 4 \bullet 3)\sqrt{3 \bullet 5 \bullet 7} = 24\sqrt{105}$$

Show Professor K that you understand how to rationalize the denominator in mathematical expressions by working the activities for this lesson.

LIFE PRINCIPLE



"A rational person calmly works through problems."

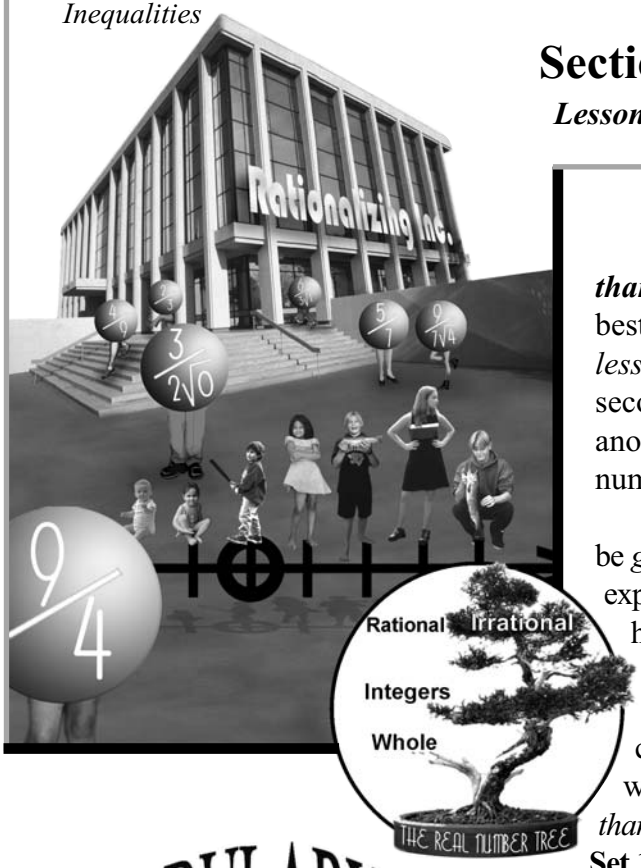
—Anonymous

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Chapter 4

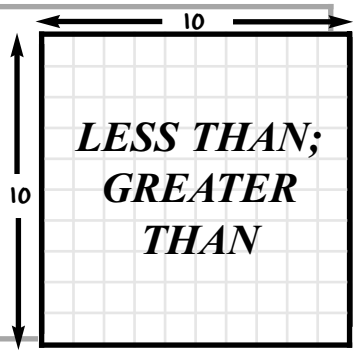
Lesson 3

Inequalities



Section 1

Lessons 1-5



You will remember that the value of one number can be ***greater than*** or ***less than*** another number. This concept is understood best by looking at a number line. A number that is *less than* another number is to the left of the second number. A number that is *greater than* another number is to the right of the second number.

Mathematical expressions can also be shown to be *greater than* or *less than* another mathematical expression. Professor K will show you in this lesson how to graph the value of an unknown in mathematical statements known as ***inequalities***.

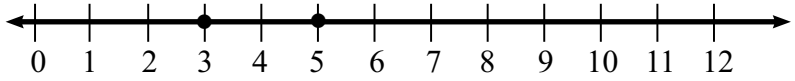
To help you understand inequalities containing mathematical expressions, Professor K will give you a quick review of *greater than* or *less than*. Professor K offers the statements in **Example Set 1** to refresh your memory.

VOCABULARY

Greater than: numbers to the right on a number line; symbol, >
Less than: numbers to the left on a number line; symbol, <
Inequality: a mathematical statement containing a *greater than* or *less than* symbol
Open circle: symbol used to mark the beginning point of the graph of the solution of a *greater* or *less than* statement on a number line

Example Set 1

a) $3 < 5$



Three is to the *left* of five on the number line; therefore, three is *less than* five.

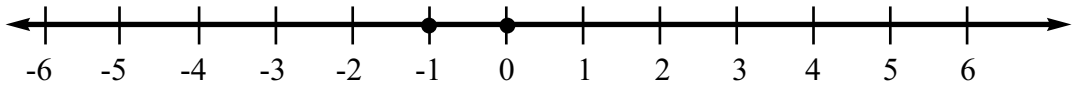
b) $45 < 50$



Forty-five is to the *left* of fifty on the number line; therefore, forty-five is *less than* fifty.

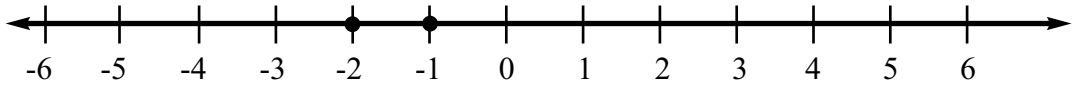
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c) $-1 < 0$



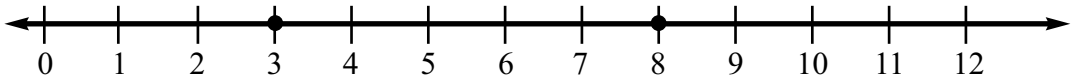
Negative one is to the *left* of zero on the number line; therefore, negative one is *less than* zero.

d) $-1 > -2$



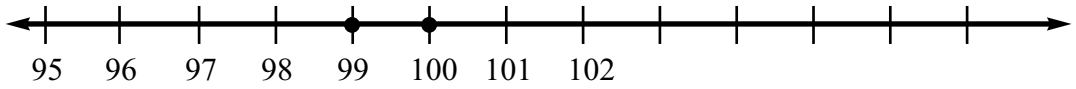
Negative one is to the *right* of negative two on the number line; therefore, negative one is *greater than* negative two.

e) $8 > 3$



Eight is to the *right* of three on the number line; therefore, eight is *greater than* three.

f) $100 > 99$



One hundred is to the *right* of ninety-nine on the number line; therefore, one hundred is *greater than* ninety-nine.

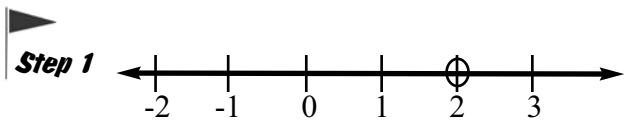
Look at **Example Set 2** as Professor K shows you how to graph an inequality statement containing an unknown.

Example Set 2

Graph the following inequality on a number line.

$x > 2$ $(x = 2)$

Look at the statement of inequality: $x > 2$. Professor K graphs the problem as if it were a statement of equality ($x = 2$) to determine the starting point of the graph.



In Step 1, Professor K draws a graph of $x = 2$ on a number line using an ***open circle*** to mark the location of the beginning point (2). Professor K uses an open circle, because x cannot equal 2.

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Professor K remembers that for a number to be greater than 2, the number must be to the right of two on a number line. Therefore, Professor K draws an arrow on the graph beginning at the open circle and pointing to the right in the direction of the value of x, which can be any number greater than, but not equal to 2. Therefore, any number to the right of 2 is a possible value of x. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the right of the 2 as shown.

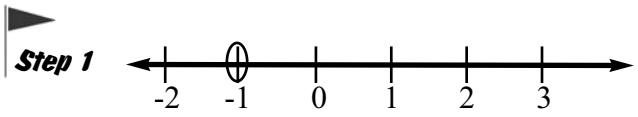
Professor K will show you how to graph inequalities that are *greater than or equal to* ($\geq$) or *less than or equal to* ($\leq$) in the next lesson. Look at **Example Set 3** as Professor K graphs another inequality.

Example Set 3

Graph the following inequality on a number line.

$x < -1$ $(x = -1)$

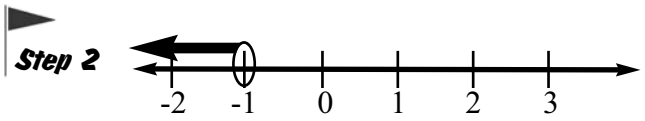
Look at the statement of inequality: $x < -1$. Professor K graphs the problem as if it were a statement of equality ($x = -1$) to determine the starting point of the graph.



In Step 1, Professor K draws a graph of $x = -1$ on a number line using an open circle to mark the location of the beginning point, -1. Professor K uses an open circle because x cannot equal -1.



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Professor K remembers that for a number to be less than -1, the number must be to the left of -1 on a number line. Therefore, Professor K draws an arrow on the graph beginning at the open circle and pointing to the left in the direction of the value of *x*, which can be any number less than, but not equal to -1. Therefore, any number to the left of -1 is a possible value of *x*. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of *x* above the number line to the left of the -1 as shown.

Notice that the symbol, O, marks the point on the number line in **Example Sets 2 and 3** where the graph begins. This symbol is an open circle because the endpoint of the graph is not included in the answer.

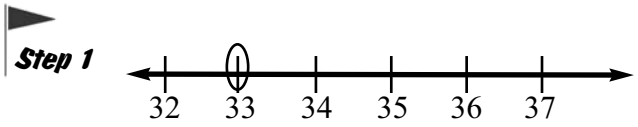
Look at **Example Set 4**, as Professor K shows you how to write an inequality statement from a word problem and graph the solution.

Example Set 4

Amanda is older than her brother. If Amanda's brother is 33 years old, write a statement of inequality that represents all the possible ages that Amanda could be, then graph the solution on a number line.

Let *x* represent Amanda's age. Then $x > 33$ represents all of Amanda's possible ages if she is older than (greater in age), but not the same age of her brother.

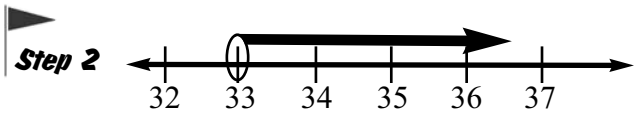
Professor K lets *x* represent Amanda's age. All of the possible ages that Amanda can be if she is older than her brother, who is 33 years of age, is represented by $x > 33$.



In Step 1, Professor K draws a graph of $x = 33$ on a number line using an open circle to mark the location of the beginning point, 33 (Amanda's brother's age). Professor K uses an open circle because *x* cannot equal 33 (Amanda and her brother are not the same age).



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Professor K remembers that for a number to be greater than 33, the number must be to the right of 33 on a number line. Therefore, Professor K draws an arrow on the graph beginning at the open circle and pointing to the right in the direction of the value of *x*, which can be any number greater than, but not equal to 33; therefore, any number to the right of 33 is a possible value of *x* (Amanda's possible age). Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of *x* above the number line to the right of the 33 as shown.

Show Professor K that you understand how to work problems involving statements of inequalities by working the activities for this lesson.

LIFE PRINCIPLE



"All men are created equal."

—United States Declaration of Independence

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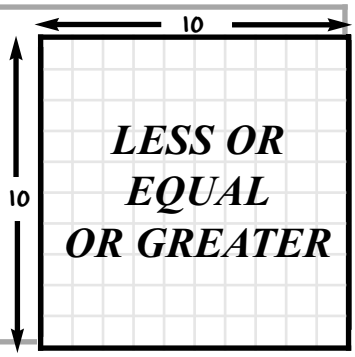
Intermediate Math Skills



Lesson 4

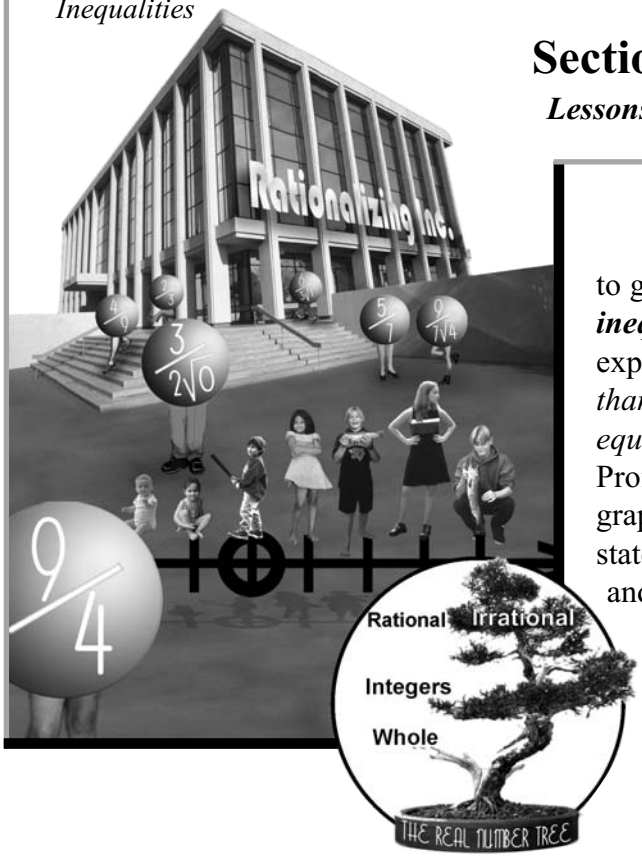
Inequalities

Chapter 4



Section 1

Lessons 1-5



In the previous lesson, Professor K showed you how to graph *greater than* and *less than* ***inequality*** statements. Mathematical expressions can also contain *greater than or equal to* and *less than or equal to* inequality symbols.



Professor K

Professor K will show you in this lesson how to graph the value of an unknown in mathematical statements containing *greater than or equal to* and *less than or equal to* inequality symbols.

You will remember the four basic symbols we have learned:

a) = means equal to
b) <, > means less than or greater than
c) ≤, ≥ means less than or equal to and greater than or equal to
d) O means the starting point on a graph relating to symbols <, >.

Look at **Example Set 1** as Professor K shows you how to graph an inequality statement containing an unknown.

VOCABULARY

Inequality: a mathematical statement containing a greater than or less than symbol
Darkened circle: ●; indicates the starting point on a line graph for an inequality when the symbols ≥ or ≤ are used

Example Set 1

Graph the following inequality on a number line.

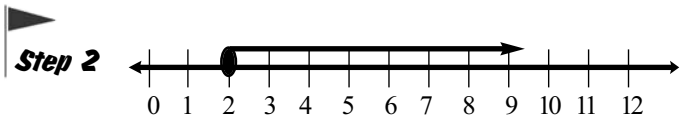
$x \geq 2$ $(x = 2)$

Looking at the statement of inequality $x \geq 2$, Professor K graphs the problem as if it were a statement of equality $(x = 2)$ to determine the starting point of the graph.

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In Step 1, Professor K draws a graph of $x = 2$ on a number line using a darkened circle to mark the location of the beginning point (2). Professor K uses a ***darkened circle***, because x can equal 2.



Professor K remembers that for a number to be greater than 2, the number must be to the right of 2 on a number line. Therefore, Professor K draws an arrow on the graph beginning at the darkened circle and pointing to the right in the direction of the value of x. The x can be any number greater than, or equal to 2, therefore, 2 and any number to the right of 2, is a possible value of x. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the right of the 2 as shown.

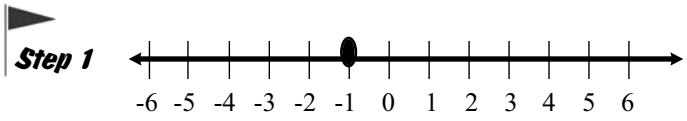
Look at **Example Set 2** as Professor K graphs another inequality

Example Set 2

Graph the following inequality on a number line.

$x \leq -1$ $(x = -1)$

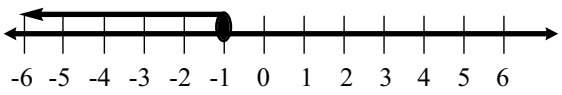
Look at the statement of inequality: $x \leq -1$. Professor K graphs the problem as if it were a statement of equality $(x = -1)$ to determine the starting point of the graph.



In Step 1, Professor K draws a graph of $x = -1$ on a number line using a darkened circle to mark the location of the beginning point (-1). Professor K uses a darkened circle, because x can equal -1.

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Step 2



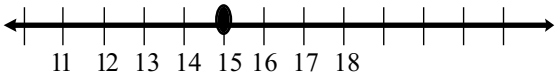
Professor K remembers that for a number to be less than -1, the number must be to the left of -1 on a number line. Therefore, Professor K draws an arrow on the graph beginning at the darkened circle and pointing to the left in the direction of the value of x. The x can be any number less than, or equal to -1; therefore, -1 or any number to the left of -1 is a possible value of x. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the left of the -1 as shown.

Look at **Example Set 3**, as Professor K shows you how to write an inequality statement from a word problem and graph the solution.

Example Set 3

Timothy is the same age or younger than his sister. If Timothy's sister is 15 years old, write a statement of inequality that represents all the possible ages that Timothy could be, then graph the solution on a number line. Let x represent Timothy's age. Then $x \leq 15$ represents all of Timothy's possible ages if he is the same age or younger than his sister (less than or equal in age).

Step 1

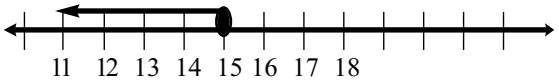


In Step 1, Professor K draws a graph of $x = 15$ on a number line using a darkened circle to mark the location of the beginning point, 15 (Timothy's sister's age). Professor K uses a darkened circle because x can equal 15 (Timothy and his sister could be the same age).



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Step 2



Professor K remembers that for a number to be less than 15, the number must be to the left of 15 on a number line. Therefore, Professor K draws an arrow on the graph beginning at the darkened circle and pointing to the left in the direction of the values of x. The x can be any number less than or equal to 15, therefore, any number to the left of 15 is a possible value of x (Timothy's possible ages). Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the left of the 15 as shown.

Show Professor K that you understand how to work problems involving statements of inequalities by working the activities for this lesson.

LIFE PRINCIPLE



"Positive or negative choices always equal positive or negative consequences."

—Ronald Johnson

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Chapter 4

Lesson 5

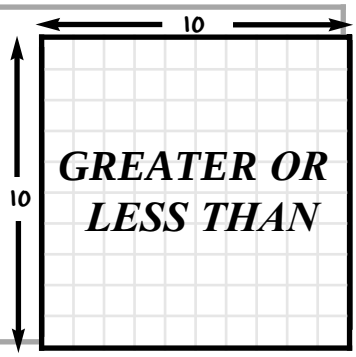
Inequalities





Section 1

Lessons 1-5



Previously, Professor K told you a story of a man who purchased apples for his children. He had to make sure each child received the same size and price of apple to be equitable with his children. The same was true for equations. You must perform the same operation to both sides of the equation to keep the equality of the mathematical expressions.

Remember, you have learned that the value of one number can be *greater than* or *less than* another number. Mathematical expressions can also be shown to be *greater than* or *less than* another mathematical expression. In Lesson 3, Professor K reviewed *greater than* or *less than*. In Lessons 3 and 4, he showed you how to graph ***inequality*** statements on a number line. Professor K will show you in this lesson how to determine the value of the unknown in mathematical inequality statements.

In a *statement of inequality*, the same basic rules apply as in *statements of equality*. Whatever operation is performed on one side of the inequality symbol, the same operation must be performed on the other side. Professor K demonstrates in **Example Set 1** how to solve an inequality using the rule shown below.



Inequality: a mathematical statement containing a greater than or less than symbol



RULE

To maintain the truth of an inequality statement, whatever operation is performed on the expression(s) on one side of the inequality symbol must also be performed to the expression(s) on the other side of the inequality symbol.

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Example Set 1

This example shows you how to solve the inequality for x. You will also learn how to graph the solution on a number line.

Step 1 $2x > 4 \quad (2x = 4)$

Look at the statement of inequality: $2x > 4$. Professor K works the problem as if it is a statement of equality ($2x = 4$) to emphasize that the procedure for solving for the unknown (x) is the same as a statement of equality.

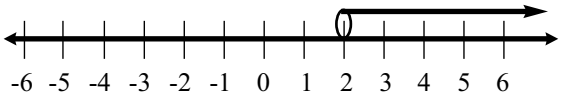
Step 2 $\frac{2x}{2} = \frac{4}{2}$

Professor K divides both sides by two according to the rule for solving statements of equality (and inequality).

Step 3 $(x = 2)$
$x > 2$

Professor K determines that the value of x is 2. Professor K now replaces the = with > to determine that the value of x is greater than 2.

Step 4



Professor K draws a graph of a number line using an open circle to mark the location of the beginning point (2). An open circle is used because x cannot equal 2. The unknown (x) represents any number greater than, but not equal to 2; therefore, any number to the right of 2 is a possible value of x. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the right of the 2 as shown.

Professor K will show you how to work more problems that are *greater than or equal to* or *less than or equal to* in the next lesson. Look at **Example Set 2** as Professor K works another inequality problem.



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Example Set 2

Solve the following inequality for x. Graph the solution on a number line.

$4x + 5 < 25$

Look at the statement of inequality, $4x + 5 < 25$, as Professor K solves the problem for x according to the above rule. Professor K will not show the statement as a statement of equality as he did in **Example Set 1**. However, Professor K still uses the procedure for solving for the unknown (x) the same as if it were a statement of equality.

Step 1 $4x + 5 - 5 < 25 - 5$
$4x + 0 < 20$

In Step 1, Professor K subtracts 5 from both sides to begin the process of solving for x by isolating it on the lhs of the equation.

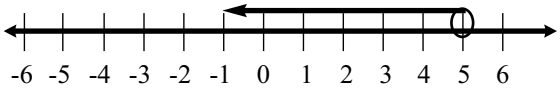
Step 2 $4x < 20$
$\frac{4x}{4} < \frac{20}{4}$

After performing the subtraction in Step 1, Professor K obtains the inequality in Step 2, $4x < 20$. Professor K now divides both sides of the inequality statement by 4 as shown.

Step 3 $x < 5$

Step 3 shows the result of completing the calculations in Step 2. Professor K determined that the value of x is less than 5.

Step 4



Professor K graphs the solution to the problem in Step 4. Notice that the graph of this problem contains an open circle at the location of the starting point, 5. A dark arrow is drawn above the number line in the direction of all the values of x less than 5. Values less than 5 are to the left of 5 on the number line, therefore the dark arrowed line points left as shown.

As in previous lessons, notice that the symbol, O, marks the point on the number line in **Example Sets 1 and 2** where the graph begins. This symbol is an open circle because the endpoint of the graph is not included in the answer.

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Example Set 3

In four years Sally will be older than her brother will be at that time. If Sally's brother will be 34 in four years, calculate all of the possible ages that Sally could be now.

Let x represent Sally's age now.
Then $x + 4$ represents Sally's age in four years.

Professor K lets x represent Sally's age now.
$x + 4$ represents Sally's age in 4 years.
Professor K notices in the problem that Sally's brother will be 34 in four years.

Step 1 Sally's age in four years is greater than her brother's age in four years.

$$x + 4 > 34.$$

Professor K uses the information above to set up the inequality $x + 4 > 34$.

Step 2

$$x + 4 - 4 > 34 - 4$$
$$x + 0 > 30$$
$$x > 30$$

Professor K begins to solve the inequality by subtracting 4 from both sides. Professor K performs the calculations and determines that Sally must be more than 30 years old.

Show Professor K that you understand how to work problems involving statements of inequalities by working the activities for this lesson.

Life Principle



"The company you keep equals the trouble you meet."

—Anonymous

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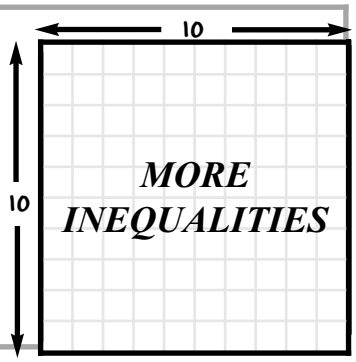
Intermediate Math Skills



Lesson 6

Inequalities

Chapter 4



Section 2

Lessons 6-10



Many statements of inequality are also equality statements, as you saw in Lesson 4. This sounds strange, but the mathematical statements are actually "or" statements. Values of numbers are either equal to another number or the value of the number is greater than or less than another number. Professor K knows that common sense and ***logic*** will tell you that something cannot be equal and unequal at the same time. Most food is served hot or cold. The measurement of an object is considered big or small, long or short.

You have reviewed the terms *greater than* or *less than* and worked mathematical expressions that include those terms. Professor K will show you in this lesson how to determine the value of unknowns in mathematical statements known as *inequalities* that also include a statement of equality. These types of statements follow the same rules as other inequalities. Look at **Example Set 1** as Professor K shows you how to work *greater than or equal to* and *less than or equal to* problems.

Remember, in a statement of inequality, even an inequality containing an *equal-to* statement, the same basic rules apply as in statements of equality. Whatever operation is performed on one side of the inequality symbol, the same operation must be performed on the other side.

VOCABULARY

Logic: thinking in an orderly fashion

Translates: repeats in another language or style

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Example Set 1

This example will show you how to solve the inequality, $4x \geq 8$ for x. You will also learn how to graph the solution on a number line.

Step 1 $4x \geq 8$ $(4x = 8)$

Look at the statement of inequality: $4x \geq 8$. Professor K ***translates*** the problem into words as follows; four times a number is greater than or equal to eight. Professor K works the problem as if it is a statement of equality $(4x = 8)$ to emphasize that the procedure for solving for the unknown, x, is the same as for a statement of equality.

Step 2 $^{4x}/_{4} = {}^{8}/_{4}$

Professor K divides both sides by four, according to the rule for solving statements of equality (and inequality).

Step 3 $(x = 2)$
$x \geq 2$

Professor K determines that the value of x is 2. Professor K now replaces the = with $\geq$ to see that the value of x is greater than or equal to 2.

Step 4



Professor K draws a graph of a number line using a darkened circle to mark the location of the beginning point, 2. A darkened circle is used because x can equal 2. The unknown (x) represents any number greater than or equal to 2; therefore, 2 and any number to the right of 2 is a possible value of x. Professor K shows this by drawing a heavy black line with an arrow in the direction of all the possible values of x above the number line to the right of the 2 as shown.

Look at **Example Set 2** as professor K works another inequality problem to help your understanding of solving inequality problems.



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Example Set 2

This example will show you how to solve the inequality $7x + 6 \leq 55$ for x. You will then learn how to graph the solution on a number line.

Step 1 $7x + 6 \leq 55$

Look at the statement of inequality: $7x + 6 \leq 55$. Professor K will not show the statement as a statement of equality as he did in **Example Set 1**. However, Professor K still uses the procedure for solving for the unknown (x) the same as if it were a statement of equality.

Step 2 $7x + 6 - 6 \leq 55 - 6$
$7x + 0 \leq 49$

In Step 2, Professor K subtracts 6 from both sides to begin the process of solving for x by isolating it on the lhs of the equation.

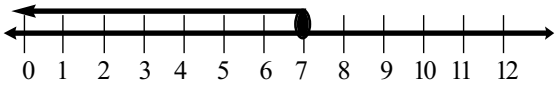
Step 3 $7x \leq 49$
$\frac{7x}{7} \leq \frac{49}{7}$

After performing the subtraction in Step 2, Professor K obtains the inequality in Step 3: $7x \leq 49$. Professor K now divides both sides of the inequality statement by 7 as shown.

Step 4 $x \leq 7$

Step 4 shows the result of completing the calculations in Step 3. Professor K determined that the value of x is less than or equal to 7.

Step 5



Professor K graphs the solution to the problem in Step 4. Notice that the graph of this problem contains a darkened circle at the location of the starting point (7) because x can be equal to 7. A dark arrow is drawn above the number line in the direction of all the values of x less than 7. Values less than 7 are to the left of 7 on the number line, therefore the dark arrowed line points left as shown.

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Example Set 3

When John places fifty more dollars in his savings account, the balance of his savings account will be greater than or equal to the balance of Jose's savings account. Jose has $110.00 in his savings account. Calculate all of the possible values for the balance of John's savings account at this time.

Let x represent the balance of John's savings account now.

Then $x + 50$ represents the balance of John's savings account later.

The balance of John's savings account after adding fifty dollars is greater than or equal to the balance of Jose's savings account.

Step 1	$x + 50 \geq 110$	Professor K uses the information above to set up the inequality, $x + 50 \geq 110$.
Step 2	$x + 50 - 50 \geq 110 - 50$ $x + 0 \geq 60$ $x \geq 60$	Professor K begins to solve the inequality by subtracting 50 from both sides. Professor K performs the calculations and determines that the balance of John's savings account must be greater than or equal to $60.00.

Show Professor K that you understand how to work problems involving *less than or equal to* and *greater than or equal to* statements by working the activities for this lesson.

Life Principle



"The greater the problem, the greater the opportunity for success."

—Anonymous

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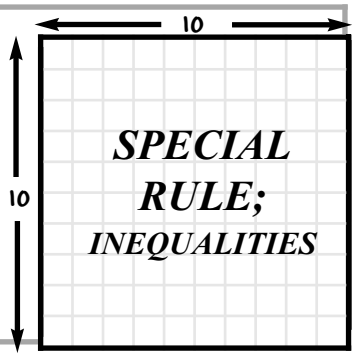
Intermediate Math Skills



Lesson 7

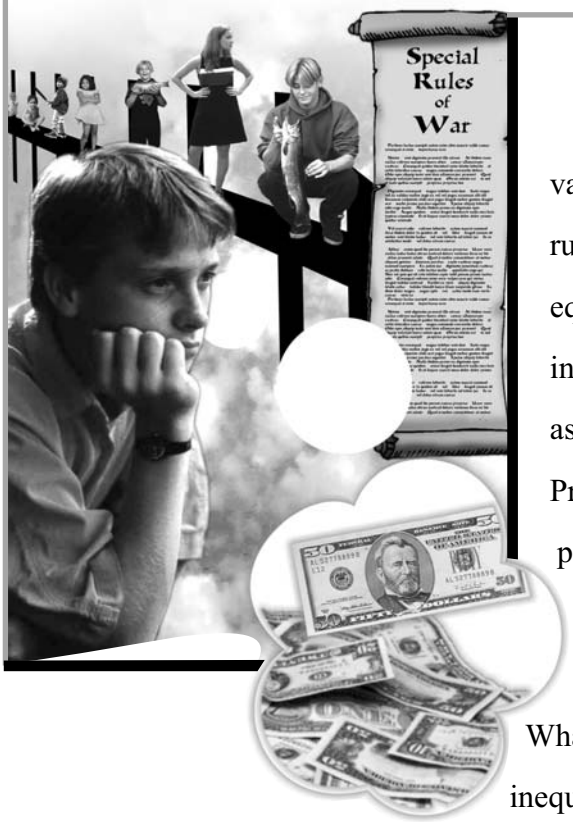
Inequalities

Chapter 4



Section 2

Lessons 6-10



In previous lessons, Professor K has shown you how to work various inequality statements using the rules associated with statements of equality. While thinking about statements of inequality, Professor K remembered a ***special rule*** associated with working such problems. Furthermore, Professor K thought of how special rules are often placed on citizens in times of war or terrorist activity.



Professor K

Remember, in a statement of inequality, the same basic rules apply as in statements of equality. Whatever operation is performed on one side of the inequality symbol, the same operation must be performed on the other side.

RULE

When an inequality is multiplied or divided by a negative number, the inequality symbol must be reversed.

VOCABULARY

Special rule: a rule used for a specific purpose; may not be true in every situation

Look at the examples as Professor K shows you how to use the rule shown above to work what he calls a special case for inequalities.

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Example Set 1

This example will show you how to solve the inequality, $-3x \geq 12$ for x.

Step 1	$-3x \geq 12$ $(-3x = 12)$	Look at the statement of inequality, $-3x \geq 12$, while Professor K translates the statement into words as follows: negative three times a number is *greater than or equal to* twelve. Professor K solves the problem as if it were a statement of equality $(-3x = 12)$ to emphasize that the procedure for solving for the unknown, x, is the same as a statement of equality.
Step 2	$^{-3x}/_{-3} = {}^{12}/_{-3}$	Professor K divides both sides by negative three, according to the rule for solving statements of equality (and inequality).
Step 3	$(x = -4)\ x \geq -4$	Professor K determines that the value of x is -4. Professor K now replaces the = with $\geq$ to see that the value of x is greater than or equal to -4.
Step 4	$-3x \geq 12$ $-3(0) \geq 12$	Professor K picks a number that is greater than -4 to check his solution to the problem. Professor K chooses $x = 0$ and substitutes it into the original statement to check his answer for accuracy.
Step 5	$0 \geq 12$ not True	Professor K determines that the solution that he obtained in not correct. The reason is the rule that Professor K forgot to follow: When an inequality is multiplied or divided by a negative number, the inequality symbol must be reversed.
Step 6	$x \leq -4$	Professor K corrects his solution to the problem by reversing the inequality symbol.
Step 7	$-3x \geq 12$ $-3(-5) \geq 12$ $15 \geq 12$ True	Professor K picks a number that is less than -4 to check his solution to the problem. Professor K chooses $x = -5$ and substitutes it into the original statement to check his answer for accuracy and determines that his solution is correct.



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Professor K points out that you should always check your solution by using the original statement of inequality to be sure that you changed (reversed) the symbol correctly. Look at **Example Sets 2 and 3** as Professor K works other inequality problems containing negative signs.

Example Set 2

Solve the following inequality for x.

$-8x - 3 \leq 69$

Step 1	$-8x - 3 + 3 \leq 69 + 3$ $-8x + 0 \leq 72$	Look at the statement of inequality: $-8x - 3 \leq 69$. Professor K will not show the statement as a statement of equality as he did in **Example Set 1**. However, Professor K still uses the procedure for solving for the unknown (x) the same as if it were a statement of equality. In Step 1, Professor K adds 3 to both sides to begin the process of solving for x by isolating it on the lhs of the equation.
Step 2	$-8x \leq 72$ $\frac{-8x}{-8} \leq \frac{72}{-8}$	After performing the addition in Step 1, Professor K obtains the inequality in Step 2: $-8x \leq 72$. Professor K now divides both sides of the inequality statement by -8 as shown.
Step 3	$x \geq -9$	Step 3 shows the result of completing the calculations in Step 2. Professor K determined that the value of x is greater than or equal to -9. Notice that Professor K reversed the inequality symbol to reflect the division by a negative number in Step 2.
Step 4	$-8x - 3 \leq 69$ $-8(0) - 3 \leq 69$ $0 - 3 \leq 69$ $-3 \leq 69$ True	Professor K checks his solution in Step 4 by choosing a number (0) that is greater than -9. Professor K chooses $x = 0$ and substitutes it into the original statement to check his answer for accuracy and determines that his solution is correct because -3 is less than 69.

Example Set 3

Solve the following inequality for x.

$\frac{5x}{-3} + 4 > 14$

Step 1	$\frac{5x}{-3} + 4 - 4 > 14 - 4$ $\frac{5x}{-3} + 0 > 10$ $\frac{5x}{-3} > 10$	Professor K begins to solve the inequality by subtracting 4 from both sides. Professor K then performs the calculations to obtain the inequality, $\frac{5x}{-3} > 10$.

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Step 2	$5x/-3 \bullet (-3) > 10 \bullet (-3)$ $5x < -30$	Professor K multiplies -3 times both sides of the statement of inequality to obtain $5x < -30$. (Professor K reversed the inequality symbol because he multiplied both sides by a negative three.)
Step 3	$x < -6$	Professor K divides both sides by 5 and performs the calculations to obtain the solution of $x < -6$.
Step 4	$^{5(-9)}/_{-3} + 4 > 14$ $15 + 4 > 14$ $19 > 14$ True	Professor K checks his solution by choosing a number (-9) that is less than -6 and placing -9 in the original inequality, then simplifying to find that the solution is correct.

Show Professor K that you understand how to work problems involving *less than or equal to* and *greater than or equal to* statements by working the activities for this lesson.

Life Principle



"Great men are willing to change as needed to accomplish their goals."

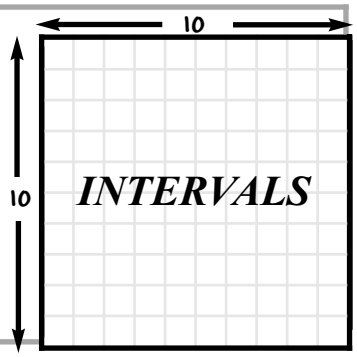
—Anonymous

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Lesson 8

Inequalities

Chapter 4



Section 2

Lessons 6-10



In previous lessons, Professor K has shown you how to calculate the value of an unknown in statements of inequalities. Often, the value of the unknown is not just greater or less than a particular number, but the unknown is greater than one number while at the same time less than another number. The value of this unknown is then between the two numbers. This is called an ***interval***.

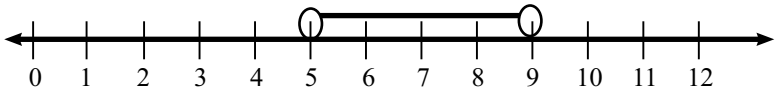
Look at the examples as Professor K shows you how to determine the interval into which an unknown falls when two inequality statements are combined.

Example Set 1

To become familiar with the terms and graphs of intervals, look at the following inequalities that define an interval.

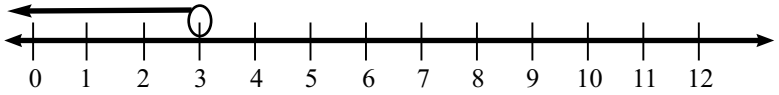
a) $5 < x < 9$

The statement, $5 < x < 9$ is called an ***open interval*** because both end points are not included in the solution.



b) $x < 3$

This statement is also an open interval, because the end point is not included in the solution, and there is no stopping point on the other end.





Interval: a set of numbers defined by an inequality statement

Open interval: an interval that does not include at least one end point or one end point does not exist (one end does not have a stopping point)

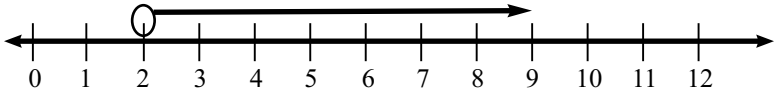
Closed interval: an interval in which both end points are included

Half-open interval: an interval in which one end is closed (end point included) and one end is open (end point not included)

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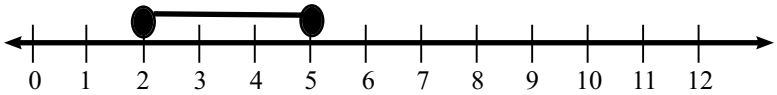
c) $x > 2$

This statement is an open interval, because the end point is not included in the solution, and there is no stopping point on the other end.



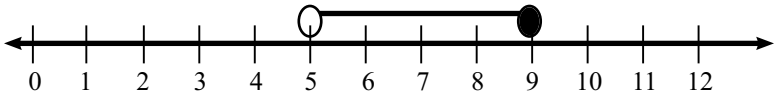
d) $2 \leq x \leq 5$

The statement, $2 \leq x \leq 5$ is called a ***closed interval*** because both end points are included in the solution.



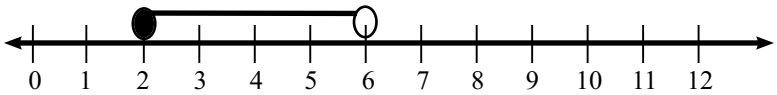
e) $5 < x \leq 9$

This statement is called a ***half-open interval***, because one end point is included in the solution and one end point is not.



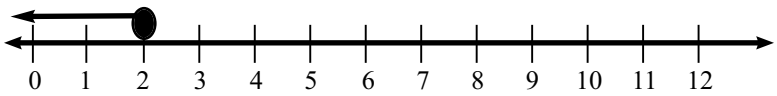
f) $2 \leq x < 6$

This statement is called a half-open interval, because one end point is included in the solution and one end point is not.



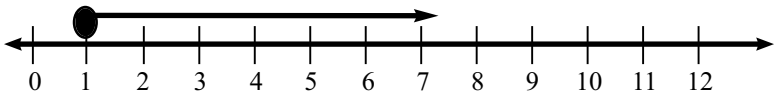
g) $x \leq 2$

This statement is called a half-open interval, because one end point is included in the solution and there is no stopping point on the other end



h) $x \geq 1$

This statement is called a half-open interval, because one end point is included in the solution and there is no stopping point on the other end.



Example Set 2

Determine if the interval shown is an open, half-open or closed interval.

a) $4 > x$

a) This is an open interval because the end points are not included.

b) $3x \leq 5$

b) This is a half-open interval because one end point is included but the interval has no stopping point.

c) $5 \leq 4x \leq 50$

c) This is a closed interval because both end points are included.

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d) $125 > 25x > 500$ d) This is an open interval because neither end point is included.

e) $-34 \geq 2x > -64$ e) This is a half-open interval because only one end point is included.

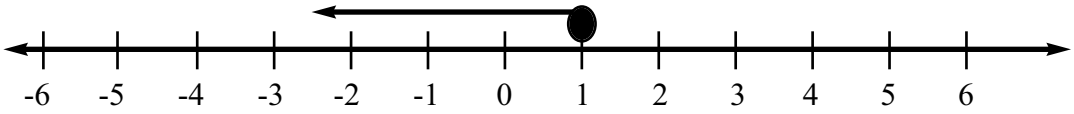
f) $64 \geq x + 2 \geq 100$ f) This is a closed interval because both end points are included.

g) $-2 < -8x - 3 \leq 69$ g) This is a half-open interval because only one end point is included.

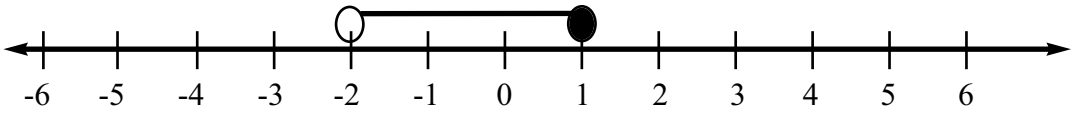
Example Set 3

Determine if the graph of the interval shown is an open, half-open or closed interval.

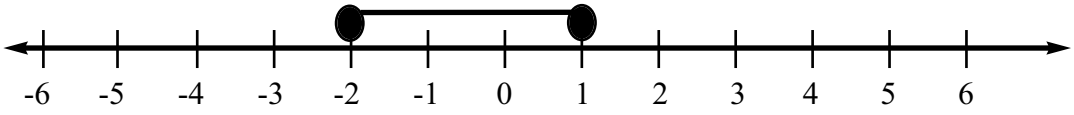
a) half-open interval



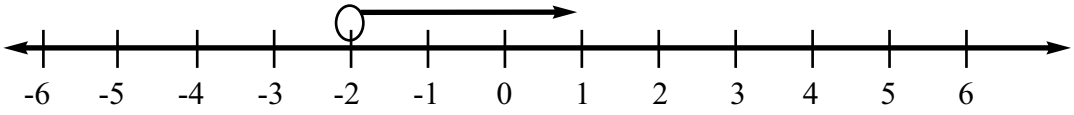
b) half-open interval



c) closed interval



d) open interval



Show Professor K that you understand how to work problems involving intervals by working the activities for this lesson.

LIFE PRINCIPLE



"Life is the interval of time between birth and death. Make it count for something."

—Anonymous

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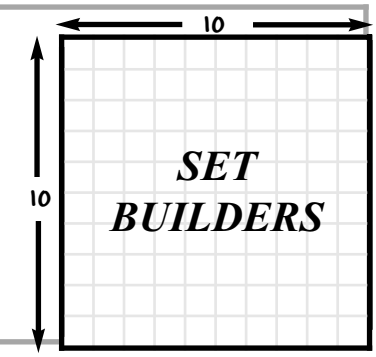
Lesson 9

Inequalities

Chapter 4

Section 2

Lessons 6-10







Professor K

Remember when Professor K discussed sets, that there was a special way to show a set. The elements of sets were placed in curly brackets, {1, 2, 3, 4}. An interval is also a set of numbers. Two methods exist in mathematics to indicate an interval: ***set-builder notation*** and interval notation. Professor K will explain set-builder notation in this lesson and interval notation in Lesson 10.

To show the solution to an inequality in set-builder notation, the inequality statement is placed within curly brackets and rewritten with the symbol "x |" in front of the inequality statement as shown in **Figure 9.1**.

Statement of inequality	Set-builder notation	How to translate notations
$x > 5$	$\{x \mid x > 5\}$	all x, such that (|), the value of x is greater than five
$2 \leq x \leq 3$	$\{x \mid 2 \leq x \leq 3\}$	all x, such that (|), two is less than or equal to the value of x and x is less than or equal to three
$x \leq 10$	$\{x \mid x \leq 10\}$	all x, such that (|), the value of x is less than or equal to ten
$4 \leq x < 7$	$\{x \mid 4 \leq x < 7\}$	all x, such that (|), four is less than or equal to the value of x and x is less than seven

Figure 9.1

VOCABULARY

Set-builder notation: a mathematical method of expressing an interval as seen on a number line; the inequality statement is put in curly brackets

{x | . . . }: means, all values of x such that . . . ; reads, all x, such that . . .

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Notice in **Figure 9.1**, that you simply place the simplified statement of inequality within a set of curly brackets.

Look at the examples as Professor K shows you how to determine the interval in which an unknown falls when two inequality statements are combined.

Example Set 1

Look at the following statements of inequality. Write each statement in set-builder notation.

a) $5 < x < 9$
set-builder notation: $\{x \mid 5 < x < 9\}$

For set-builder notation, Professor K places $5 < x < 9$ in curly brackets along with the mathematical shorthand for "all x such that." 5 is less than x and at the same time, x is less than 9.

b) $x < 3$
set-builder notation: $\{x \mid x < 3\}$

For set-builder notation, Professor K places $x < 3$ in curly brackets along with the mathematical shorthand for "all x such that."

c) $2 \leq x \leq 5$
set-builder notation: $\{x \mid 2 \leq x \leq 5\}$

For set-builder notation, Professor K places $2 \leq x \leq 5$ in curly brackets along with the mathematical shorthand for "all x such that." 2 is less than or equal to x and at the same time, x is less than or equal to 5.

In **Example Set 2** Professor K changes the English words into set-builder notation.

Example Set 2

a) all x such that 15 is greater than x and x is greater than -5

a) $\{x \mid 15 > x > -5\}$

b) all x such that -2 is less than x and x is less than 8

b) $\{x \mid -2 < x < 8\}$

c) all x such that 5 is less than or equal to x and x is less than or equal to 50

c) $\{x \mid 5 \leq x \leq 50\}$

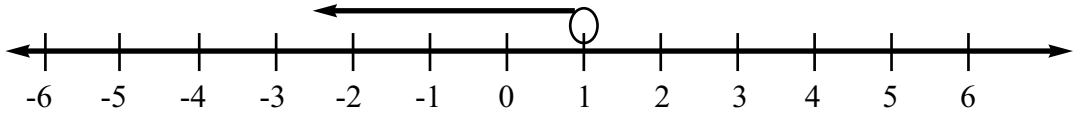
In **Example Set 3** Professor K expresses in set-builder notation the solution shown in each graph.



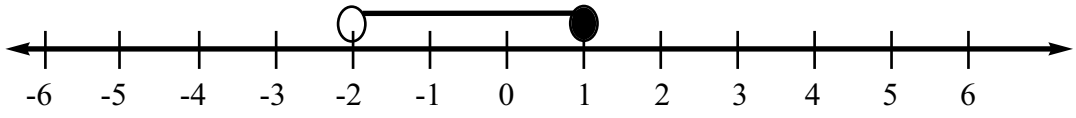
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Example Set 3

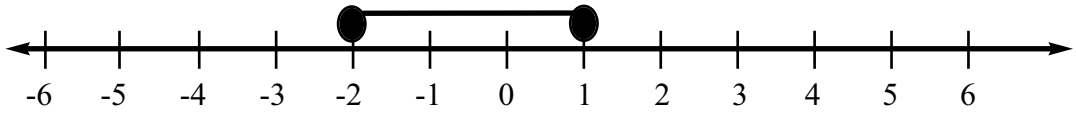
a) $\{x \mid x < 1\}$



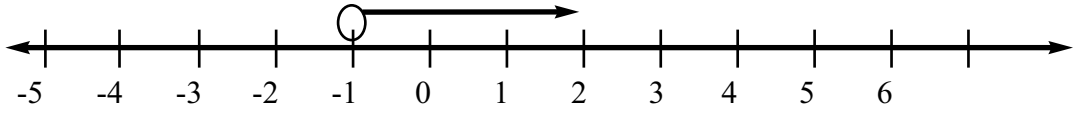
b) $\{x \mid -2 < x \leq 1\}$



c) $\{x \mid -2 \leq x \leq 1\}$



d) $\{x \mid x > -1\}$



Show Professor K that you understand how to write intervals in set-builder notation by working the activities for this lesson.

LIFE PRINCIPLE



"A wise man builds on a firm foundation."

—Hebrew Principle

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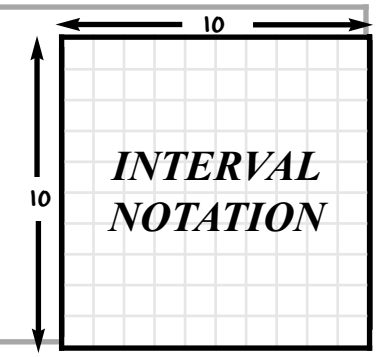
Lesson 10

Inequalities

Chapter 4

Section 2

Lessons 6-10





In the previous lesson, Professor K showed you how to express the solution to intervals in set-builder notation. The statement of inequality was placed in curly brackets. Professor K will explain ***interval notation*** in this lesson while reviewing set-builder notation with you so that you can compare the two methods of expressing the interval.

To show the solution to an inequality in set-builder notation, the inequality statement is placed within curly brackets as shown in **Figure 10.1**. Also **Figure 10.1** shows the other method of indicating an interval, *interval notation*. Notice that **Figure 10.1** is similar to **Figure 9.1** in Lesson 9 with a column to show interval notation.

	Statement of inequality	Set-builder notation	Interval notation
(a)	$x > 5$	$\{x \mid x > 5\}$	$(5, \infty)$
(b)	$2 \le x \le 3$	$\{x \mid 2 \le x \le 3\}$	$[2, 3]$
(c)	$x \le 10$	$\{x \mid x \le 10\}$	$(-\infty, 10]$
(d)	$4 \le x < 7$	$\{x \mid 4 \le x < 7\}$	$[4, 7)$

Figure 10.1

Notice in **Figure 10.1**, that the simplified statement of inequality is placed within a set of curly brackets in set-builder notation. For interval notation, the end points of the inequality are written separated by a comma. A parenthesis or a square bracket is placed before and after the set of end points depending on the following conditions. If the first end point is closed or contained in the inequality ($\ge$ or $\le$), a square bracket is placed in front of the end point. If the first end point is open or not contained in the inequality ($<$ or $>$), a parenthesis is placed before the end point. The same conditions apply to the second end point with the appropriate symbol being placed after the second end point.

VOCABULARY

Interval notation: a solution set with the endpoints of a statement put into parentheses or square brackets depending on whether its endpoint is open or closed

Infinity symbol: ∞; looks like a figure 8 on its side

Indefinitely: without stopping; forever

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Notice further in **Figure 10.1** that the ***infinity symbol*** (∞) is used when the solution continues ***indefinitely*** in a particular direction.

Look at the examples as Professor K shows you how to determine the interval in which an unknown falls when two inequality statements are combined. Notice that these examples are the same as in the previous lesson, but Professor K has added interval notation to the expression of the solutions.

Example Set 1

Look at the following statements of inequality. Write each statement in set-builder and interval notation.

a) $5 < x < 9$
set-builder notation: $\{x \mid 5 < x < 9\}$
interval notation: $(5, 9)$

For set-builder notation, Professor K places $5 < x < 9$ in curly brackets along with the mathematical shorthand for "all x such that." He then places the 5 and 9 in parentheses as shown to present the answer in interval notation.

b) $x < 3$
set-builder notation: $\{x \mid x < 3\}$
interval notation: $(-\infty, 3)$

For set-builder notation, Professor K places $x < 3$ in curly brackets along with the mathematical shorthand for "all x such that." He then places the symbol, $-\infty$ and 3 in parentheses as shown to present the answer in interval notation.

c) $2 \leq x \leq 5$
set-builder notation: $\{x \mid 2 \leq x \leq 5\}$
interval notation: $[2, 5]$

For set-builder notation, Professor K places $2 \leq x \leq 5$ in curly brackets along with the mathematical shorthand for "all x such that." He then places the symbol, 2 and 5 in square brackets as shown to present the answer in interval notation.

In **Example Set 2**, Professor K changes the English words into interval notation.

Example Set 2

a) all x such that 15 is greater than x and x is greater than -5 — a) $(-5, 15)$

b) all x such that -2 is less than x and x is less than 8 — b) $(-2, 8)$

c) all x such that 5 is less than or equal to x and x is less than or equal to 50 — c) $[5, 50]$

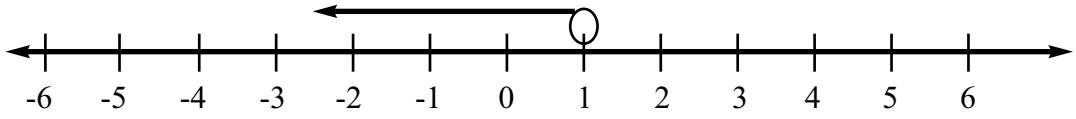


In **Example Set 3** Professor K expresses in interval notation the solution shown in each graph.

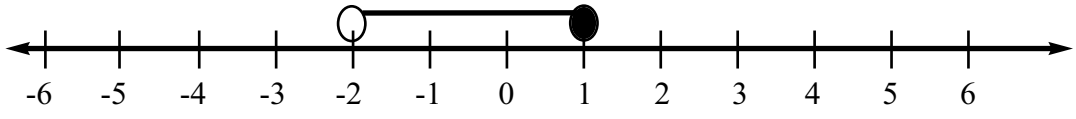
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Example Set 3

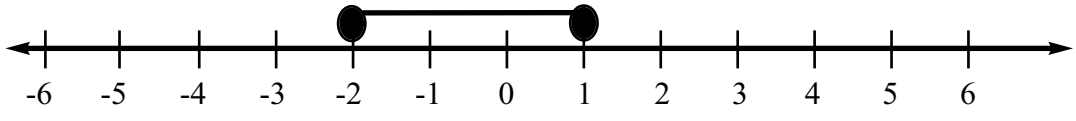
a) $(-\infty, 1)$



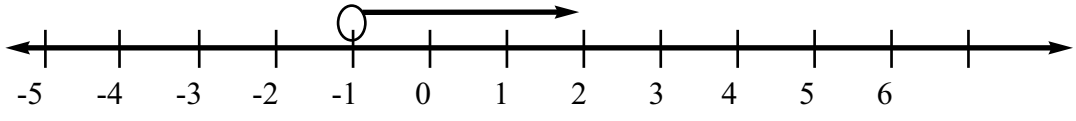
b) (-2, 1]



c) [-2, 1]



d) $(-1, \infty)$



Show Professor K that you understand how to write intervals in set-builder and interval notation by working the activities for this lesson.

LIFE PRINCIPLE



"Morality consists of drawing behavioral lines we refuse to cross"

—N.S. Johnson

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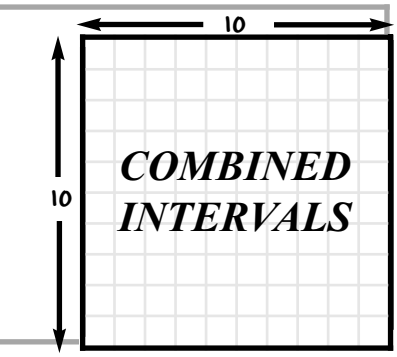
Lesson 11

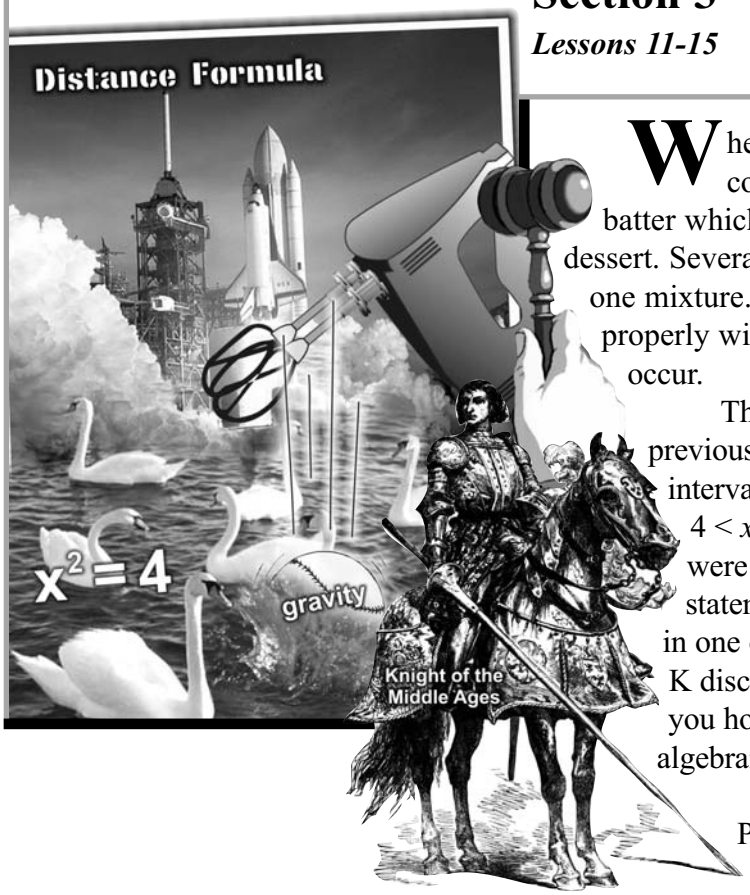
Inequalities

Chapter 4

Section 3

Lessons 11-15





When your mother bakes a cake, she combines the ingredients to make the batter which, when baked, becomes a delicious dessert. Several ingredients are combined to form one mixture. Only when all the parts are mixed properly will a good tasting result (solution) occur.

The same is true for inequalities. In previous lessons you have been introduced to intervals that have been combined. Example: $4 < x \leq 9$. Two statements of inequality were combined to make one larger statement. You can solve such inequalities in one of two ways. In this lesson, Professor K discusses those two methods and shows you how to solve ***combined intervals*** algebraically and by graphing.

Look at the examples as Professor K shows you how to determine the solution of combined inequality statements.

VOCABULARY

Combined intervals: two inequality statements combined into one statement

Reasoning: thinking through something to a logical conclusion

Example Set 1

Solve the following combined statements of inequality for x. Express the solution in set builder and interval notation.

$12 \leq 3x \leq 18$

Step 1

$12 \leq 3x$

$3x \leq 18$

Professor K separates this combined statement of inequality into two statements of inequality as shown in Step 1.

Step 2

$\frac{12}{3} \leq \frac{3x}{3}$

$\frac{3x}{3} \leq \frac{18}{3}$

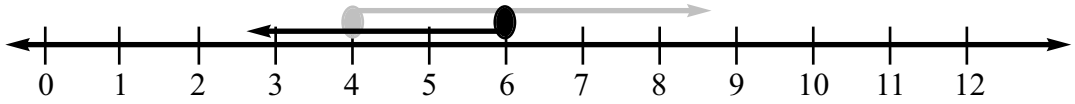
Professor K now divides both sides of each statement by 3 as shown in Step 2.

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Step 3 $4 \leq x$
$x \leq 6$
$4 \leq x \leq 6$

Professor K performs the calculations and obtains the inequalities in Step 3. Professor K can determine the solution by ***reasoning***. If 4 is less than or equal to x, and at the same time x is less than or equal to 6, then the value of x must be between and including 4 and 6. In other words, Professor K combined the two separate statements into one statement again.

Step 4



$\{x \mid 4 \leq x \leq 6\}$ or $[4, 6]$

In Step 4, Professor K shows you how to graph the separate statements of inequality on the same number line to find the solution to the combined statement. Notice that the graph of $4 \leq x$ is shown in gray, and the graph of $x \leq 6$ is shown in black. Professor K looks for the portion of the graph that is shaded twice (both gray and black). He sees that that portion is between and including 4 and 6. This portion of the graph is the solution to the combined statement of inequality.

In **Example Set 1**, Professor K showed you how to separate, work and graph a combined inequality to determine the solution. In **Example Set 2**, Professor K will show you how to determine the solution without separating the combined inequality into two separate inequalities.



Example Set 2

Solve the following combined statement of inequality for x. Express the solution in set builder and interval notation.

$16 < 4x \leq 32$

Step 1 $\frac{16}{4} < \frac{4x}{4} \leq \frac{32}{4}$

In Step 1, Professor K divides every term by 4 as shown. Remember, what you do to one, you must do to all.

Step 2 $4 < x \leq 8$

Professor K performs the calculations and determines the solution in Step 2.

Step 3 $\{x \mid 4 < x \leq 8\}$ or $(4, 8]$

The solution is written in set-builder and interval notation in Step 3.

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Example Set 3

Solve the following combined statement of inequality for x. Express the solution in set builder and interval notation.

$25 > 3x + 4 \geq 13$

Step 1	$25 - 4 > 3x + 4 - 4 \geq 13 - 4$ $21 > 3x \geq 9$	In Step 1, Professor K subtracts 4 from all 3 parts of the combined inequality statement. Remember, what you do to one, you must do to all.
Step 2	$^{21}/_{3} > {}^{3x}/_{3} \geq {}^{9}/_{3}$	In Step 2, Professor K divides every term by 3 as shown. Remember, what you do to one, you must do to all.
Step 3	$7 > x \geq 3$	Professor K performs the calculations and determines the solution in Step 3.
Step 4	$\{x \mid 7 > x \geq 3\}$ or $[3, 7)$	The solution is written in set-builder and interval notation in Step 4.

Remember that interval notation looks just like the graph would on the number line with brackets and parentheses and smaller values to the left.

Show Professor K that you understand how to work problems involving intervals by working the activities for this lesson.

LIFE PRINCIPLE



"When combined, wisdom and hard work spell success."

—Anonymous

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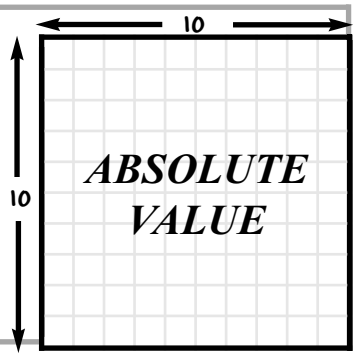
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Lesson 12

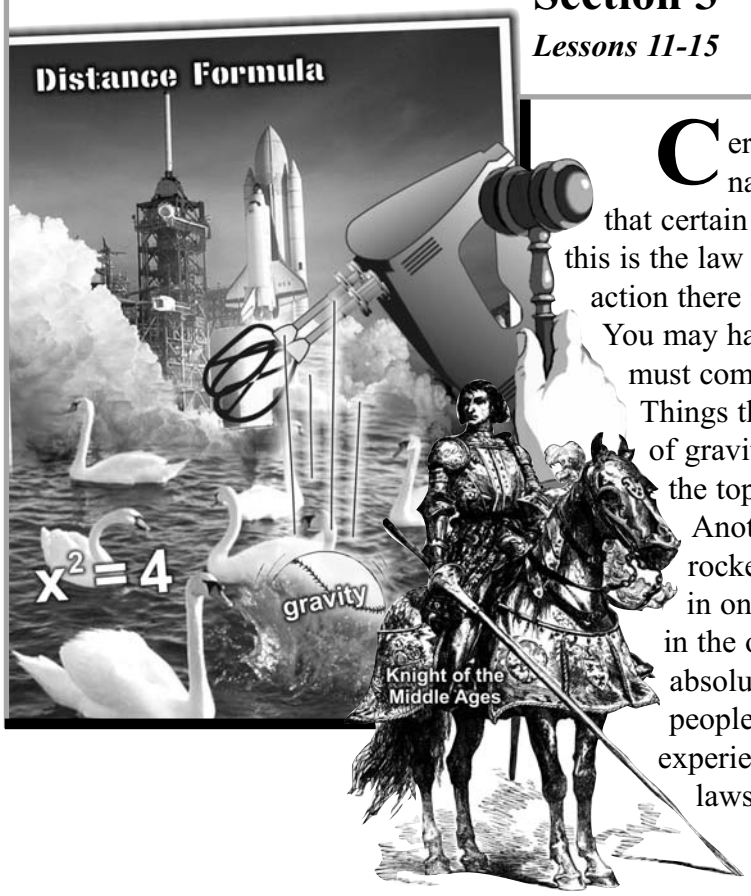
Absolute Value

Chapter 4



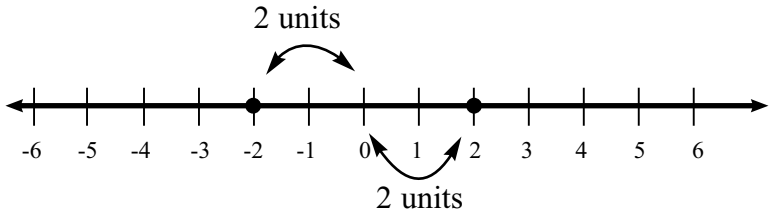
Section 3

Lessons 11-15



Certain concepts and laws that exist in nature are called ***absolutes***. This means that certain rules are always true. An example of this is the law in physics that states, "for every action there is an equal and opposite reaction." You may have heard the saying, "what goes up, must come down." *Up* and *down* are opposites. Things that go up, must come down because of gravity. For example, a ball dropped from the top of a building will fall to the ground. Another absolute is illustrated when a rocket blasts off. The thrust of the engines in one direction cause the rocket to move in the opposite direction. In society, absolutes in relationships exist between people and their choices. A person will experience consequences according to the laws of cause and effect.

The ***magnitude*** or numerical value of a number is the distance of the number from zero on a number line. Negative 2 and positive 2 are both 2 units from the origin (zero) on the number line.



Negative 2 is to the left of zero, or in the negative direction. Positive 2 is to the right of zero, or in the positive direction. The sign of the number determines direction; the magnitude of the number is its value or distance from zero. ***Absolute value*** is the magnitude of a number; or more simply, the positive value of a number. The absolute value of 2 is *2*.

VOCABULARY

Absolute: something that is always true; unquestionable
Magnitude: the distance of a number from zero on the number line
Absolute value: the magnitude, numerical or positive value of a number

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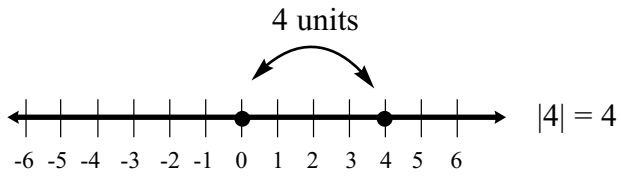
The absolute value of -2 is *2*. Both 2 and -2 have a magnitude of 2, because both numbers are located two units from zero. Look at **Example Set 1** as Professor K shows you how to determine the absolute value of numbers written within the grouping symbol for absolute value, | |.



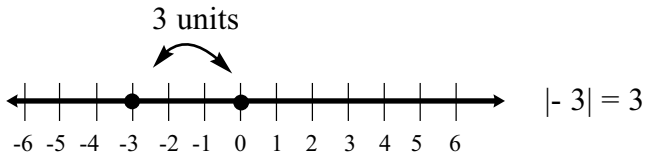
Example Set 1

Determine the absolute value of the following numbers.

a) |4| = ? Professor K determines that the absolute value of 4 is *4*, because 4 is four units from zero.



b) |-3| = ? Professor K determines that the absolute value of -3 is *3*, because -3 is three units from zero.



In **Example Set 1**, Professor K showed you that |4| = 4 and |-3| = 3 simply by looking at the distance each number was from zero. In **Example Set 2**, Professor K will determine absolute value by looking only at the magnitude of the numbers.

Example Set 2

Determine the absolute value of the following numbers.

a) |-5| = 5 Professor K determines that the absolute value of -5 is *5*, because +5 is the value of the magnitude of -5.

b) |10| = 10 Professor K determines that the absolute value of 10 is *10*, because +10 is the value of the magnitude of 10.

c) |-37| = 37 Professor K determines that the absolute value of -37 is *37*, because +37 is the value of the magnitude of -37.

d) |400| = 400 Professor K determines that the absolute value of 400 is *400*, because +400 is the value of the magnitude of 400.

e) |-1| = 1 Professor K determines that the absolute value of -1 is *1*, because +1 is the value of the magnitude of -1.

f) |0| = 0 Professor K determines that the absolute value of 0 is *0*, because +0 is the value of the magnitude of 0.

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Show Professor K that you understand how to work absolute value problems by working the activities for this lesson.

LIFE PRINCIPLE



"An absolute of great value is truth."

—Anonymous

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|7|

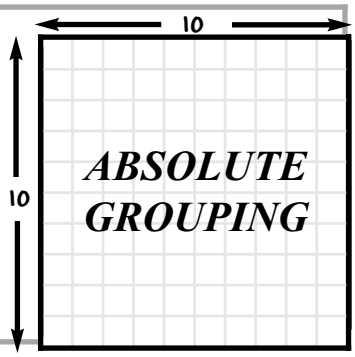
Lesson 13

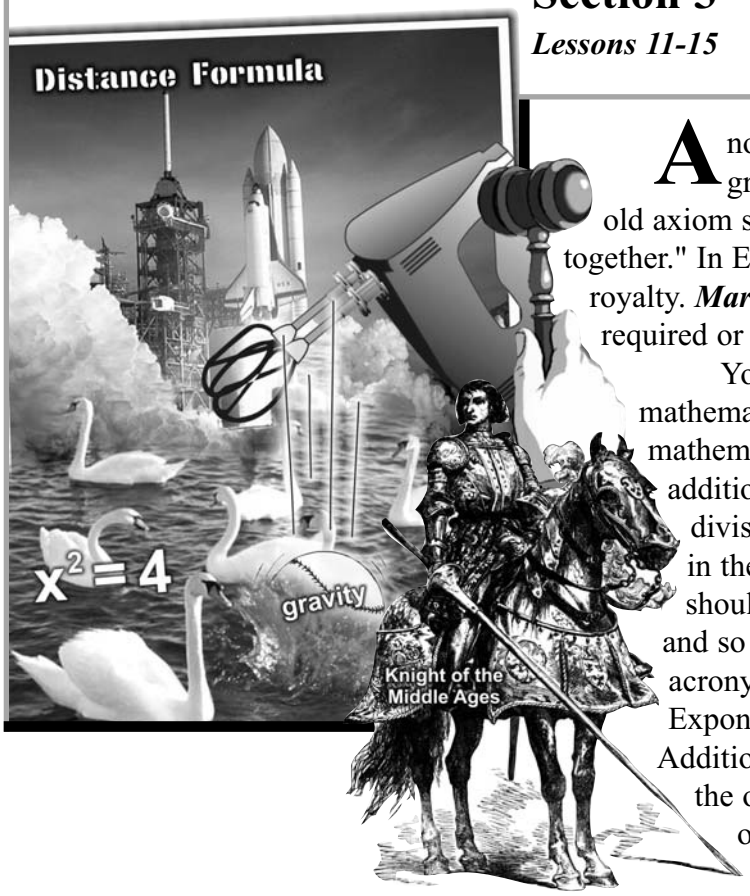
Absolute Value

Chapter 4

Section 3

Lessons 11-15





Another absolute is that people are grouped by the friends they keep. An old axiom states, "birds of a feather flock together." In Europe, ***royalty*** is expected to marry royalty. ***Marriage*** within certain groups is required or expected.

You have learned that within mathematics, numbers are grouped using mathematical properties for the purpose of addition, subtraction, multiplication and division. Grouping symbols are used to aid in the process of ordering which operation should be accomplished first, second, third and so forth. You have already learned the acronym, PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) as the proper order for the operations in simplifying a numerical or algebraic expression.

You may recall that parentheses and brackets are the most frequently used grouping symbols in mathematical expressions. The symbol for the absolute value of a number, | |, is also considered a grouping symbol.

In **Example Set 1**, Professor K shows you how the symbol for absolute value, | |, is actually considered a special grouping symbol.

VOCABULARY

Royalty: persons in the lineage of a king
Marriage: civil and spiritual union of a man and a woman

Example Set 1

Determine the absolute value of the following numbers.

a) $2\,|3| = 2 \bullet 3 = 6$ Professor K determines that the absolute value of 3 is *3*. Then he multiplies 2 times 3 to obtain 6 as the answer of 2 times the absolute value of 3.

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b) $4\ |-8| = 4 \bullet 8 = 32$

Professor K determines that the absolute value of -8 is *8*. Then he multiplies 4 times 8 to obtain 32 as the answer to 4 times the absolute value of -8.

c) $-\ |22| = -\ (22) = -22$

Professor K determines that the absolute value of 22 is *22*. Then he multiplies the negative sign which means -1 times 22 to obtain -22 as the answer to the negative of the absolute value of 22.

d) $|25 - 5| = |20| = 20$

Professor K determines that the absolute value of 25 - 5 is *20*. Professor K performs the operation of subtraction within the grouping symbol of the absolute value to obtain |20| which equals 20 as the final answer.

e) $|-45 + 25| = |-20| = 20$

Professor K determines that the absolute value of -45 + 25 is *20*. Professor K performs the operation of addition within the grouping symbol of the absolute value to obtain |-20| which equals 20 as the final answer.

f) $-\ |13 - 20| = -\ |-7| = -(7) = -7$

Professor K determines that the negative of the absolute value of 13 - 20 is *-7*. Professor K performs the operation of subtraction within the grouping symbol of the absolute value to obtain |-7| = 7. Then Professor K must multiply the negative sign times the answer to obtain -7 as the final answer to -|13 - 20|.

In **Example Set 2**, Professor K shows you how to simplify more problems containing absolute value symbols using PEMDAS.

Example Set 2

Determine the absolute value of the following numbers.

a) $2\ |4| + 5 = (2 \bullet 4) + 5 =$
$8 + 5 = 13$

Professor K determines that the absolute value of 4 is *4*. Then he multiplies 2 times 4 to obtain 8 as the answer to 2 times the absolute value of 4. Professor K then adds 8 to 5 to obtain 13 as the final answer.

b) $3\ |-8 - 2| = 3\ |-10| =$
$3 \bullet 10 = 30$



Using PEMDAS Professor K performs the operation within the absolute grouping symbol first. Professor K performs the operation, |-8 - 2| to obtain |-10|. Professor K determines that the absolute value of -10 is *10*. Then he multiplies 3 times 10 to obtain 30 as the answer to 3 times the absolute value of -8 minus 2.

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c) - |22| + 25 = - (22) + 25 =
-22 + 25 = 3

Professor K determines that the absolute value of 22 is *22*. Then he multiplies the negative sign, which means -1 times 22 to obtain -22. Remember, you must perform multiplication before addition. Professor K adds -22 to 25 to obtain 3 as the answer to the negative of the absolute value of 22 plus 25.

d) 2 |25 - 5| + 4 = 2 |20| + 4
2(20) + 4 = 40 + 4 = 44

Professor K determines that the absolute value of 25 -5 is *20*. Professor K performs the operation of subtraction within the grouping symbol of the absolute value to obtain |20| and then Professor K multiplies 2 times 20 to obtain 40. Professor K then adds 40 and 4 to obtain 44 as the final answer.

e) |-45 + 25| + |-10| =
|-20| + |-10| =
20 + 10 = 30

Professor K determines that the absolute value of -45 + 25 by performing the operation of addition within the first grouping symbol of the absolute value to obtain |-20| which equals 20. Professor K then determines the absolute value of -10 is *10*. Professor K adds these two numbers to obtain 20 + 10 = 30 as the final answer.

Show Professor K that you understand how to work absolute value problems by working the activities for this lesson.

Life Principle



"If you don't want anyone
to know, don't do it."
—Chinese Proverb

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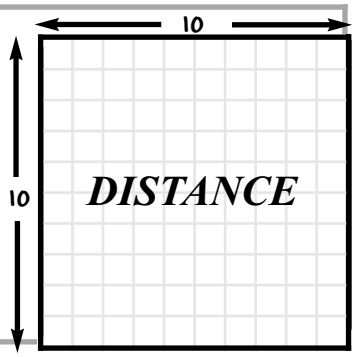
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Lesson 14

Algebra

Chapter 4



Section 3

Lessons 11-15



Finding the length of a line in the Cartesian coordinate system can be accomplished using a formula known as the ***distance formula***. Professor K shows you the distance formula below.

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The *distance formula* states that the distance between two points is found by calculating the square root of the sum of the square of the difference of the ***abscissas*** (*x*-values) of each end point and the square of the difference of the ***ordinates*** of the end points (*y*-values).

This formula is a derivation of the Pythagorean Theorem, $c^2 = a^2 + b^2$. The *a* represents the length of the distance along the x-axis (the difference of the abscissas). The *b* represents the length of the distance along the y-axis (the difference of the ordinates). Look at **Figure 14.1** to better understand this formula.

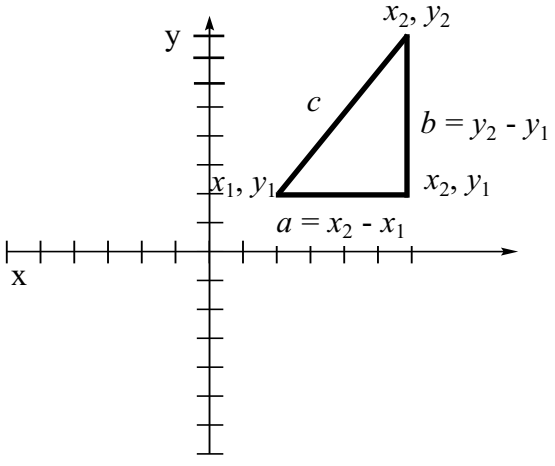


Figure 14.1

VOCABULARY

Distance formula: formula used to calculate the length of a line segment from the coordinates of the end points
Abscissas: x-axis values
Ordinates: y-axis values

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To find the value of c in the Pythagorean Theorem, $c^2 = a^2 + b^2$, you must calculate the square root of each side of the equation. Therefore, $c = \sqrt{a^2 + b^2}$, and substituting $a = (x_2 - x_1)$ and $b = (y_2 - y_1)$, yields the equation, $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

In this lesson, Professor K will show you how to calculate the length of a line segment from the coordinates of the endpoints of that line segment. Look at the examples as Professor K shows you how to determine the solution of distance problems.



Example Set 1

Calculate the distance between the following points using the distance formula.

a) (4, 3), (6, 2)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(4-6)^2 + (3-2)^2}$

$\sqrt{(-2)^2 + (1)^2}$

$\sqrt{4+1}$

$\sqrt{5}$

Professor K places the abscissas and ordinates of the two points into the distance formula: $x_2 = 4$, $x_1 = 6$, $y_2 = 3$, $y_1 = 2$. Professor K points out that it does not matter which point you choose as (x_1, y_1) and (x_2, y_2). However, you must be sure to keep the respective abscissa and ordinate in their proper locations. Professor K performs the subtractions within the parentheses $(4 - 6)^2 = (-2)^2$ and $(3 - 2)^2 = 1^2$. Professor K then squares the results. $(-2)^2 = -2 \bullet -2 = 4$ and $1^2 = 1 \bullet 1 = 1$. He then adds the squares $4 + 1 = 5$, to obtain $\sqrt{5}$ units as the distance between the points (4, 3) and (6, 2).

b) (-2, 1), (3, 1)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(-2-3)^2 + (1-1)^2}$

$\sqrt{(-5)^2 + (0)^2}$

$\sqrt{25+0}$

$\sqrt{25} = 5$

Professor K places the abscissas and ordinates of the two points into the distance formula: $x_2 = -2$, $x_1 = 3$, $y_2 = 1$, $y_1 = 1$. Professor K points out that it does not matter which point you choose as (x_1, y_1) and (x_2, y_2). However, you must be sure to keep the respective abscissa and ordinate in their proper locations. Professor K performs the subtractions within the parentheses $(-2 - 3)^2 = (-5)^2$ and $(1 - 1)^2 = 0^2$. Professor K then squares the results. $(-5)^2 = -5 \bullet -5 = 25$ and $0^2 = 0 \bullet 0 = 0$. He then adds the squares $25 + 0 = 25$, to obtain $\sqrt{25} = 5$ units as the distance between the points (-2, 1) and (3, 1).

In **Example Set 1**, Professor K showed you how to calculate the distance of a line segment between two points when given the coordinates of those points. In **Example Set 2**, Professor K will show you how to calculate the distance of two more line segments to make sure that you understand how to use the distance formula.

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Example Set 2

Calculate the distance between the following points using the distance formula.

a) (-4, 3), (6, -2)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(-4-6)^2 + (3-(-)2)^2}$

$\sqrt{(-10)^2 + (5)^2}$

$\sqrt{100+25}$

$\sqrt{125}$

$\sqrt{25 \bullet 5} = \sqrt{25} \bullet \sqrt{5}$

$5\sqrt{5}$

Professor K places the abscissas and ordinates of the two points into the distance formula: $x_2 = -4$, $x_1 = 6$, $y_2 = 3$, $y_1 = -2$. Professor K points out that it does not matter which point you choose as (x_1, y_1) and (x_2, y_2). However, you must be sure to keep the respective abscissa and ordinate in their proper locations. Professor K performs the subtractions within the parentheses $(-4 - 6)^2 = (-10)^2$ and $(3 - (-2))^2 = 5^2$. Professor K then squares the results: $(-10)^2 = -10 \bullet -10 = 100$ and $5^2 = 5 \bullet 5 = 25$. He then adds the squares 100 + 25 = 125, to obtain units as the distance between the points (-4, 3) and (6, -2). This solution, however, is not in reduced or proper form because one of the factors of 125 is a perfect square. Therefore, Professor K can factor 125 into 25 • 5. He can then calculate the square root of 25 to be 5. The solution in proper form becomes $5\sqrt{5}$.

b) (-2, 2), (3, 3)

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(-2-3)^2 + (2-3)^2}$

$\sqrt{(-5)^2 + (-1)^2}$

$\sqrt{25+1}$

$\sqrt{26}$

Professor K places the abscissas and ordinates of the two points into the distance formula: $x_2 = -2$, $x_1 = 3$, $y_2 = 2$, $y_1 = 3$. Professor K points out that it does not matter which point you choose as (x_1, y_1) and (x_2, y_2). However, you must be sure to keep the respective abscissa and ordinate in their proper locations. Professor K performs the subtractions within the parentheses: $(-2 - 3)^2 = (-5)^2$ and $(2 - 3)^2 = (-1)^2$. Professor K then squares the results: $(-5)^2 = -5 \bullet -5 = 25$ and $(-1)^2 = -1 \bullet -1 = 1$. He then adds the squares 25 + 1 = 26, to obtain $\sqrt{26}$ units as the distance between the points (-2, 2) and (3, 3). Notice that this solution is in proper form because there are no factors of 26 that are perfect squares.

After reading the Life Principle on the next page, show Professor K that you understand how to work problems using the distance formula by working the activities for this lesson.



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LIFE PRINCIPLE



"You can go farther by coordinating your efforts with others."

—Anonymous

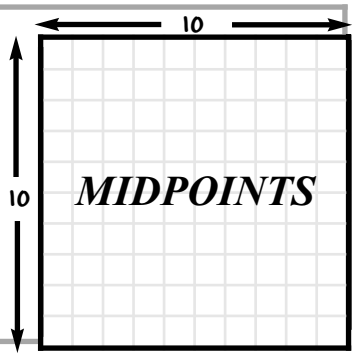
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Lesson 15

Algebra

Chapter 4



Section 3

Lessons 11-15





Historians refer to the period of time from about 500 to 1500 A.D. as the ***Middle Ages***, the middle 1000 years between the year 1 A.D. and the year 2000 A.D. In the life of an individual, the ages of 40 years old to 60 years old are considered ***middle age***. The middle of something refers to one-half of the time, journey or experience. Line segments also have a middle or can be divided into two equal pieces. Mathematicians refer to this point as the ***midpoint***, meaning that there is as much of the line segment on one side of the midpoint as on the other side of the midpoint. When a line is defined by its endpoints, which are labeled by ordered pairs, a formula exists to calculate the coordinates of the ordered pair that names the midpoint.

Professor K will present the formula to you in this lesson and show you how to use it.

The ordered pair that names the midpoint of a line segment with endpoints, (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Look at **Example Set 1** as Professor K uses the midpoint formula to determine the ordered pair that names the midpoint of the line segment between the two points given.

VOCABULARY

Middle Ages: period of time between about 500 and 1500 A.D.; also called the *Dark Ages*
Middle age: period of a man's life between the ages of 40 and 60 years of age
Midpoint: the point in the middle of a line segment

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Example Set 1

Determine the coordinates of the midpoint of the line segment between the points (2, 4) and (4, 6).

Step 1 (2, 4), (4, 6)
$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

In Step 1, Professor K writes the ordered pairs for the endpoints and the midpoint formula.

Step 2 $\frac{x_2 + x_1}{2} = \frac{4+2}{2} = \frac{6}{2} = 3$

In Step 2, Professor K calculates the x-coordinate of the midpoint by substituting the values of the x-coordinates of the endpoints into the midpoint formula as shown. Professor K performs the calculation and determines that the x-coordinate of the midpoint is *3*.

Step 3 $\frac{y_2 + y_1}{2} = \frac{6+4}{2} = \frac{10}{2} = 5$

In Step 3, Professor K calculates the y-coordinate of the midpoint by substituting the values of the y-coordinates of the endpoints into the midpoint formula as shown. Professor K performs the calculation and determines that the y-coordinate of the midpoint is *5*.

Step 4 (3, 5)

In Step 4, Professor K now places the solutions to the midpoint formula into an ordered pair that names the midpoint of the line segment between the points (2, 4) and (4, 6).

Example Set 2

Determine the coordinates of the midpoint of the line segment between the points (4, 8) and (12, 20).

Step 1 (4, 8), (12, 20)
$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

In Step 1, Professor K writes the ordered pair for the endpoints and the midpoint formula.

Step 2 $\frac{x_2 + x_1}{2} = \frac{12+4}{2} = \frac{16}{2} = 8$

In Step 2, Professor K calculates the x-coordinate of the midpoint by substituting the values of the x-coordinates of the endpoints into the midpoint formula as shown. Professor K performs the calculation and determines that the x-coordinate of the midpoint is *8*.

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Step 3 $\frac{y_2 + y_1}{2} = \frac{20+8}{2} = \frac{28}{2} = 14$

In Step 3, Professor K calculates the y-coordinate of the midpoint by substituting the values of the y-coordinates of the endpoints into the midpoint formula as shown. Professor K performs the calculation and determines that the y-coordinate of the midpoint is 14.

Step 4 (8, 14)

In Step 4, Professor K now places the solutions to the midpoint formula into an ordered pair that names the midpoint of the line segment between the points (4, 8) and (12, 20).

Show Professor K that you understand how to work problems involving finding midpoints by working the activities for this lesson.

Life Principle



"Always put off until tomorrow what you shouldn't do at all."

—Morris Mandel

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Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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